# Some Remarks on the Working Distance in Macro Photography

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When working in the macro range, it often happens that one suspects the world being bewitched. No matter how hard one tries, the image just won't focus. What are we doing wrong? One has not taken into account that „macro“ does not mean „arbitrarily close“ . In fact, to obtain a sharp, real image consider this:

> For every focal length, there is a minimum working distance.

What is this *minimum working distance*? This question can be answered sufficiently precisely with the help of the model of *Geometric Optics*[1]. Geometric optics is a physical model which describes the propagation with approximated, simple equations and ideas: Energy transport takes place by rays which can be graphically imagined as straight lines. A lens can be conceived of a very thin sheet $\Sigma$ which contains the entire refractive power of the lens (shaded blue). From each pixel a bundle of rays emerge. In this bundle three prime rays can be identified, whose course through the lens can be predicted without calculations.

1. Behind the lens any incident axis-parallel ray passes through the focal point F. In fact, these rays define the position of the focal point F and thus the focal length $f$ of a lens.

2. The straight path of an incident ray crossing the lens center H is not altered by the lens. This ray is called >main ray< .

3. An incident ray, which passes through the front focal point, runs parallel to the axis behind the lens. This is obviously the inversion of property 1.

These relations are illustrated by Figure 1. According to our perception, which is influenced by the direction of writing, the sequence of events in a graphic representation evolves from left to right. Accordingly one imagines the progression optical rays from left to right. Inspired by the symmetry embedded in properties 1 to 3 one concludes that the reverse evolution should be valid in exactly the same way. Therefore: The course

[1] cf. Eugene Hecht, *Optics, 2nd edition*, Addison-Wesley, Reading 1974, p. 128 ff

of optical rays is reversible. The distinction between >object< and >image< is basically artificial and illustrates only a particular and ephemeral perspective of the situation. situation. In our case this situation consists in a slide which for us constitutes the object and its image on the camera sensor.

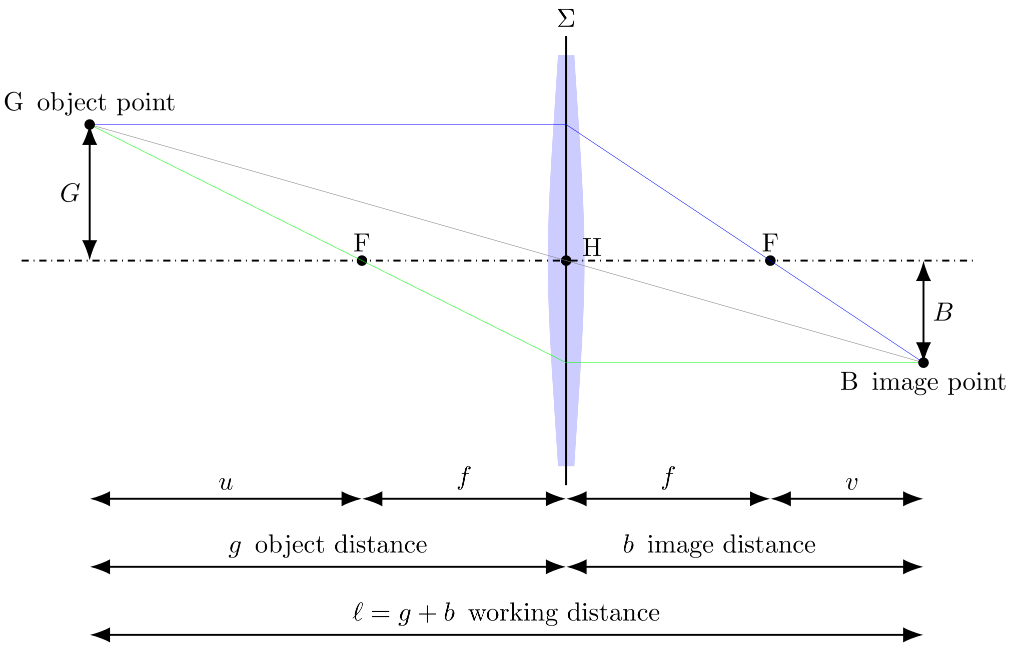


Figure 1: The imaging situation from the *Geometric Optics* point of view. The point F denotes the focal point, also called focus. A lens has two focal points, a front one and a rear one. Correspondingly, the distance $f$ denotes the focal length. The distances $G$, respectively $B$ represent the size of the object and of its image respectively.

## Minimum Working Distance

From the geometry of this figure, Newton's imaging equation can be derived

$$\frac{1}{f} = \frac{1}{g} + \frac{1}{b} \quad . \tag{1}$$

The formula tells us the following: If we take a photo of an object at a distance $g$ using a camera with a lens of focal length $f$, then the distance of the image sensor to the lens is given and set according to this formula. In addition, we recognize again that $g$ and $b$ are interchangeable.

From Figure 1 or the equation (**??**) it can be deduced that the working distance $\ell$ increases as the object point G moves away from the lens. Conversely, the image width $b$ and thus the working distance $\ell$ increases, the closer the object is to the focal point F.

Note: If the object is located between the focal point F and the lens, no real image exists any more. The ray construction then shows that the intersection point B of the special rays 1–3 lies on the same side as the object point G: B in this case is a virtual image which also appears larger than the object - the lens works as a magnifying glass.

Back to the problem: Consider $\ell$ when the image is far right, and when the object far left. In both cases we observe a large working distance $\ell$. Hence, somewhere in between there has to be a minimum $\ell$, i.e. minimum working distance. Because of the symmetry of the ray paths $1-3$ this minimum would presumably have to lie at $g=b$. If we insert this into (1), we get

$$g = b = 2f\,, \quad \text{respectively} \quad \ell = 4f\,. \tag{2}$$

For the time being, this is a conjecture based on the graphical interpretation of the ray construction in Figure 1. Can this be substantiated mathematically?

So: formula (1) contains 2 variables, $g$ and $b$, and a number $f$, which stands for the focal length of the lens used, so $f$ is known and fixed. We are interested in the working distance $\ell = g+b$. So we need an expression that expresses $\ell$ as a function of a distance, be it $g$ or $b$. For this we transfer $\ell$ into the formula (1), replacing e.g. $b$ by $b = \ell - g$ and obtain

$$\frac{1}{f} = \frac{1}{g} + \frac{1}{\ell - g} \tag{3}$$

$$\frac{1}{f} = \frac{\ell}{g(\ell - g)} \tag{4}$$

We solve for $\ell$ and obtain so the function we are looking for $\ell = \ell(g)$:

$$f\ell = g(\ell - g) \tag{5}$$

$$= g\ell - g^2 \tag{6}$$

$$\ell(g) = \frac{g^2}{g - f} \tag{7}$$

We may obtain the minimum of the function $\ell(g)$ by its graphical evaluation. However, typically the minimum of a function is found by by deriving $\ell(g)$ and setting the derivative to zero:

$$\frac{\mathrm{d}\ell(g)}{\mathrm{d}g} = 0 = \frac{\mathrm{d}}{\mathrm{d}g}\left(\frac{g^2}{g - f}\right) \tag{8}$$

according rule for quotients:

$$0 = \frac{2g(g-f) - g^2}{(g-f)^2} \tag{9}$$

$$0 = \frac{g(g-2f)}{(g-f)^2} \tag{10}$$

The quotient (10) vanishes, when the numerator vanishes. Since an object distance $g$ of zero is not a meaningful option the bracket $(g - 2f)$ must be zero, thus

$$g = 2f \quad \text{inserted in (1) results in} \tag{11}$$

$$b = 2f \quad \text{therefore} \tag{12}$$

$$\ell = 4f \tag{13}$$

which proves the assumption.

Via various considerations we have now found that the minimum working distance $\ell$ amounts to four focal lengths $f$. If we photograph with a macro lens of focal length $f = 55$ mm, below the working distance $\ell = 220$ mm a real image can not be achieved. We will come back to this further below. Before that, we will briefly consider image scale.

## Image Scale

The image scale $x$ results from the comparison of the object size $G$ with the image size $B$. As figure 1 illustrates, for ray 2 the intercept theorem holds

$$x = \frac{G}{B} = \frac{g}{b}\,. \tag{14}$$

Therefore, if at the minimum working distance we have $g = b = 2f$, then the magnification amounts to $x = 1$, the image is the same size as the object.

We have defined $x$ here to match the focus ring specifications on most macro lenses: $x$ denotes the factor by which the object is larger than the image on the camera sensor. Let us now solve (14) to obtain $g$ and let us set $g = bx$ into the imaging equation (1). Then we get

$$\frac{1}{f} = \frac{1}{bx} + \frac{1}{b} = \frac{1+x}{bx} \tag{15}$$

$$bx = f(x+1) \tag{16}$$

$$b = f + \frac{f}{x} = f + v \quad \text{(cf. Figure 1)} \tag{17}$$

Therefore:

$$v = \frac{f}{x} \tag{18}$$

The equvalent calculation for $g$ yields

$$u = fx \tag{19}$$

The distances $u$, $v$ between the focus F and the object, resp. the image, are directly related to the image scale $x$, where the focal length $f$ defines the length scale. Thus, the working distance $\ell$ of a lens of focal length $f$ can be represented as a function of the image scale $x$:

$$\ell = fx + f + f + f/x = f(2 + x + 1/x)\,. \tag{20}$$

## Geometric optics of real lenses – Practical aspects

A real objective, such as the Micro-Nikkor 55 mm 1:2.8 with its six-lens design (cf. figure 2) can certainly not be not be considered a thin lens. Of course, it would be desirable

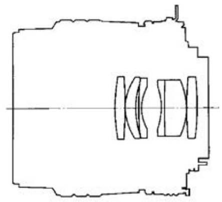

Figure 2: Structure of the Micro-Nikkor 55 mm f/2.8. (Image source: Nikon user manual)

to be able to use simple, plausible considerations and formulas to plan a photographic situation also for real lenses. With a little trick, it is possible to apply the above considerations to real lenses. We remember: A lens is modeled as a thin sheet $\Sigma$, in which the whole refractive power of the lens is concentrated; cf. figure 1. The trick is to split this lens sheet $\Sigma$ in the middle. Thus one obtains two „half-sheets“, which are shown in Figure 3 as >main planes< $\Sigma$ and $\Sigma'$, and which are separated by the distance $d$.

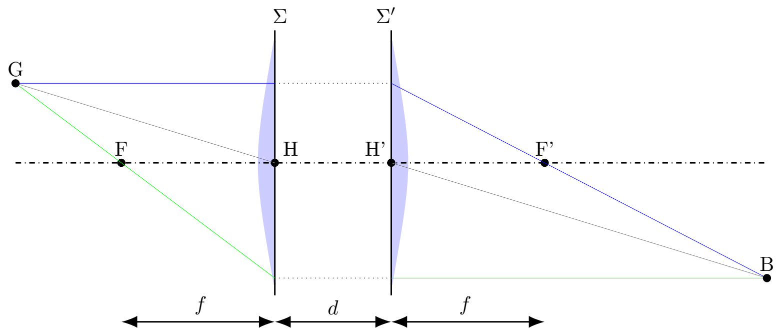


Figure 3: Modeling of a real lens in geometrical optics. The refractive power of the lens is now thought to be contained in a system consisting of two main planes, one $\Sigma$ facing the object and one $\Sigma'$ facing the image, which are separated by a kind of „Niemandsland“ of width $d$. The ray path to the left and right of the main planes follows the rules 1 – 3. One visualizes image 1 cut along $\Sigma$ and glued together again after shifting them apart by distance $d$. Accordingly, the working distance $\ell$ increases by the “objective thickness“ $d$.

However, it is not that each main plane carries half of the objective refractive power! Then one would have to construct the ray path between them. No, the ray transfer between the main planes by definition takes place parallel to the axis. A ray hitting the main plane $\Sigma$ at at a distance $r$, leaves the lens at main plane $\Sigma'$ in the same distance $r$.

In this sense, the structure ($\Sigma$–$d$–$\Sigma$) must be understood as a surrogate which reduces the complexity of an optical system to aspects essential for practice.

How does one find the location of the principal planes of a real objective? By ray tracing, when the construction of the lens is known. Alternatively, we can determine the position experimentally. As first determine the position of the focal points F and F' by letting propagate an „infinitely" distant object once from the object side and then from the image side. With a real lens it is clear where the object side and where the image (sensor) side is. In Fig. 3 the object side is shown on the left and the image side on the right. In a second step, a close object is imaged: G, B in Figure 3. We now send from G a main ray parallel to the axis in towards the lens, drawn in blue in Figure 3. This ray leaves the objective in the direction of the image-side focal point F'; Now we aim a second ray from G toward the object-side focal point F, drawn in green in Figure 3, in the direction of the objective. Behind the objective, the two rays intersect at the image point B. If now the points F, F' and G are localized, the position of the main planes can be constructed according to figure 3 and thus the distance $d$ is determined.

Practically, this distance can also be determined with a yardstick, e.g. in this way: One images a plane object, determines the scale $x$ and calculate the working distance according to (20). In addition, the working distance is measured with a yardstick. Each camera carries a mark with the position of the sensor plane for this purpose. The difference of measured minus calculated distance is the sought "objective lens thickness" $d$.

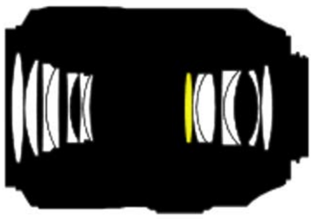

Figure 4: Optical construction of the AF-S Micro-Nikkor 105 mm f/2.8G ED; yellow: Lens made of ED glass. (image source: Nikon sales brochure).

When photographing with modern autofocus lenses of longer focal length, such as the AF-S Micro-Nikkor 105 mm 1:2.8G ED, one wonders about the working distance $\ell$: I measured a working distance of $\ell = 316\,\text{mm}$ with this lens at image scale $x = 1$. Since at imaging scale $x = 1$ the working distance is $\ell = 4f + d$ (thin lens model), the focal length can be at most $f = 316/4 = 79\,\text{mm}$, instead of 105 mm. In addition I observed that the distance between apex of the front objective lens and the sensor is only 160 mm. Both facts together mean that the image-side principal plane must lie outside, in front of the objective, and the nominal focal length of 105 mm at imaging scale of 1:1 is less than 79 mm.

Possibly an explanation can be found in the technical data of the lens and in Figure 4. The optical construction of the lens is clearly divided into two parts, systems. The left system can be easily identified as a telephoto construction consisting of the front lens group with converging function, followed by a lens group with diverging function. In principle this system shifts the main planes beyond the front lens towards the object. The right part of the structure must house the elements for image stabilization and integrate the focusing function. The latter is solved in the right system as in a zoom construction, i. e. the focal length is adjusted according to the imaging scale. As a consequence the nominal focal length, here 105 mm, is only valid at the infinity setting. Since the outer construction length of the lens does not change when focusing close, the focal length is shortened. In sum, this is a common concept of how to construct a lens with internal focusing. The effort with a total of 14 lenses is due to the large distance range from $\infty$ to scale 1:1.

Therefore, when working in the macro regime, the design of the lens must also be considered. Lenses with classic focusing, in which the entire lens stack is shifted, maintain the specified focal length even at close range. Lenses with internal focusing, on the other hand, may have larger deviations from the specified focal length at close distances.

With classical focusing lenses, the close-up range can be extended with extension rings, as for example with the PK-13 ring on the Micro-Nikkor 55 mm 1:2.8. However, extension rings are not recommended for internally focused lenses. This would put these lenses outside the range for which the design was optimized, and visible image errors are then very likely.

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